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Amending the w_* Velocity Scale for Surface Layer, Entrainment Zone, and Baroclinic Shear in Mixed Forced/Free Turbulent Convection

by

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We amend the free convective mixed layer velocity scale, w_* to w_{*s} to include mechanical turbulence from forced convection. w_{*s} has discrete surface layer, entrainment zone, and baroclinic components and requires the length scales: z_0, L, Δh, z_i, and z_b, a newly defined baroclinic length. To close the system we offer simple models for Δh, the entrainment zone depth. The Δh models show realistic diurnal hysteresis, using a critical bulk Richardson number which exceeds unity.

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INTRODUCTION

Deardorff's (1972) similarity theory for convective boundary layers (CBL)s suggests that CBL parameters are controlled by buoyancy, $\beta \equiv g/\Theta$, (where g is gravitational acceleration and Θ is potential temperature), surface temperature flux, $\overline{w'\Theta'_0}$, height z above the surface, and z_i, the height of the inversion base which marks the top of the mixing-layer (ML). The velocity, temperature, and humidity scales: w_*, u_*^{ML}, Θ_*^{ML}, q_*^{ML}, formed from these parameters are used in universal relationships for the gradients and fluxes of mean and turbulent quantities in the CBL. The first of these, the convective turbulence scaling velocity

$$w_* \equiv (\beta z_i \overline{w'\Theta'_0})^{1/3} \quad , \qquad (1)$$

determines much of CBL physics, including diffusion. Along with $\overline{w'\Theta'_0}$, w_* forms the basis for the remaining parameters used to scale the heat, moisture, and momentum, as well as passive scalar fluxes. However, w_* neglects mechanical turbulence due to shear and is only strictly valid for free convection induced by surface heating. Yet, w_* is used often in mixed forced/free convection cases for lack of alternatives. Here we seek w_{*_s}, an amendment to w_*, which includes mechanical turbulence contributions under mixed forced/free convection conditions. We assume mechanical generation comes from three sources: the barotropic, horizontally homogeneous surface layer (SL), barotropic boundary layer entrainment zone (EZ), and a baroclinity (BC) component. Beyond the z and z_i length scales, we propose that mixed forced/free convection also depends on the SL roughness length, z_0, SL Obukhov length, $L \equiv -u_*^3\Theta/(gk\overline{w'\Theta'_0})$ (where u_* is friction velocity and $k \simeq 0.4$ is the von Karmen constant), EZ thickness, Δh, and z_b, a baroclinic length scale which we define below. We model these mechanical sources, using relevant ratios of the length scales, thereby fulfilling similarity requirements.

APPROACH

We outline the basic approach before discussing details. For non-baroclinic, horizontally homogeneous, steady state cases, the vertical profiles of the buoyancy and momentum fluxes must be linear, if the shapes of the vertical profiles of potential temperature and wind speed are to remain static. That is,

$$\beta\overline{w'\Theta'_z} \simeq \beta\overline{w'\Theta'_0}(1 - (1 + a_1)z/z_i) \quad , \qquad (2a)$$

$$-\overline{u_i'w'_z} \simeq -\overline{u_i'w'_0}(1 - (1 + a_2)z/z_i) \quad , \qquad (2b)$$

where $a_1 = -\overline{w'\theta'}_{zi}/\overline{w'\theta'}_0$ and $a_2 = \overline{u_i'w'}_{zi}/\overline{u_i'w'}_0$ (see fig. 1). The subscripts "zi" and "0" refer to conditions at the inversion base and surface. $-\overline{u'w'}_0 \equiv u_*^2$, $\overline{w'\theta'}_{zi} \sim We\Delta\theta$, and $-\overline{u_i'w'}_{zi} \simeq \Delta u_i We$, where the CBL entrainment rate, $We = z_{i,t}$ and $\Delta\theta$ is the potential temperature jump across the EZ. The subscript ",t" denotes the partial derivative with respect to time. Such notation will refer to spatial and temporal derivatives in general. The Betts (1974) correction, $\overline{w'\theta'}_{zi} \simeq (\Delta\theta - \Gamma_u \Delta h)We$, hardly affects eqn. 2a since Kamada (1988a) showed that the much smaller interfacial thickness, l_m, must replace the EZ depth, Δh. For lack of better information, we assume that above z_i the barotropic flux profiles are linear with height up to $H \equiv z_i + \Delta h$, the top of the boundary layer. I.e.,

$$\beta\overline{w'\theta'}_z \simeq \beta\overline{w'\theta'}_{zi}\left(1 - \frac{z - z_i}{\Delta h}\right) \quad , \tag{3a}$$

$$-\overline{u_i'w'}_z \simeq -\overline{u_i'w'}_{zi}\left(1 - \frac{z - z_i}{\Delta h}\right) \quad . \tag{3b}$$

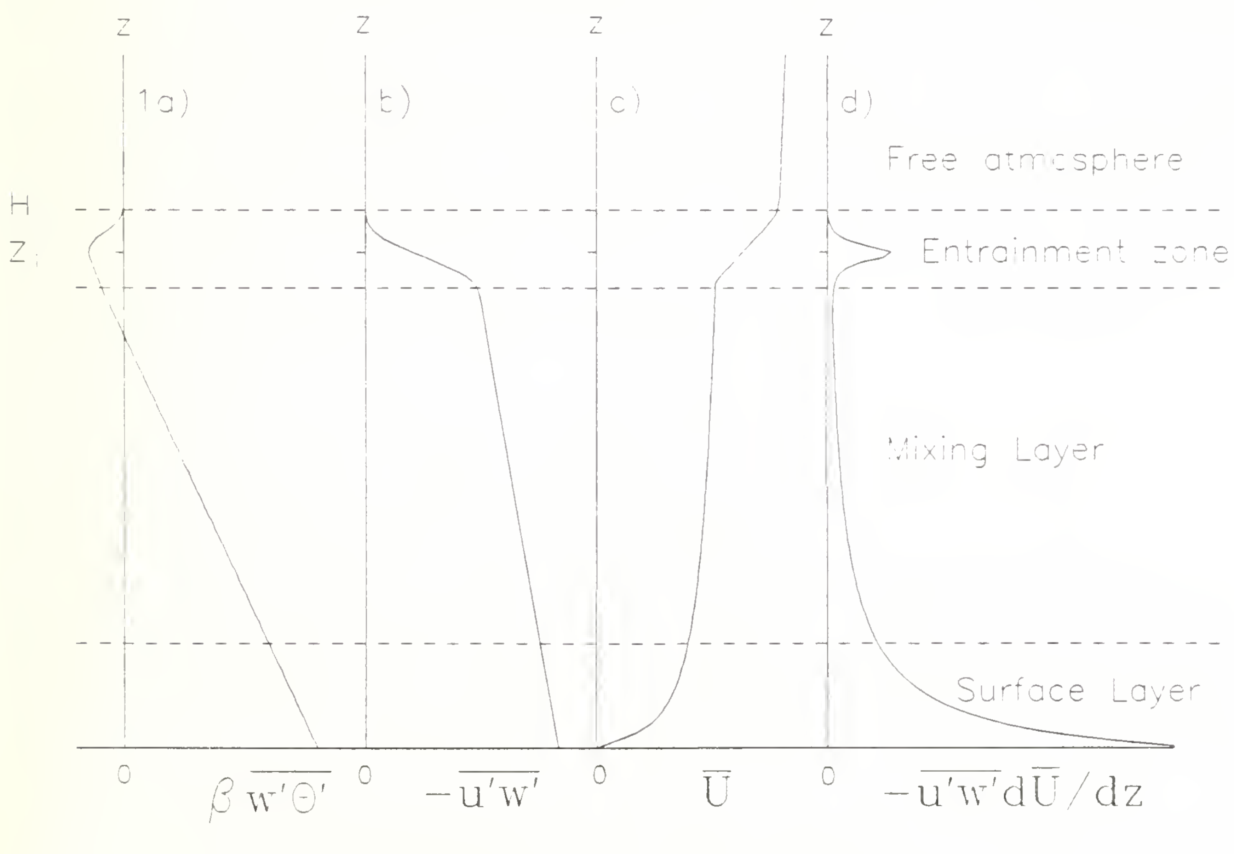
Using eqns. (1), (2a), and (3a), the vertically integrated buoyancy flux or total CBL buoyancy production rate of turbulence kinetic energy (TKE) can be written to show that the purely free convective turbulent scaling velocity is proportional to the cube root of the boundary layer integrated TKE production rate,

$$\int_{z0}^{H} \beta\overline{w'\theta'}_z \, \partial z \simeq \cap \, w_*^3 \quad , \quad \text{where } \cap \equiv \frac{1 - a_1(1 + \alpha)}{2} \quad , \tag{4}$$

and $\alpha \equiv \Delta h/z_i$. $\cap \simeq 0.4$ for mature CBLs because under such conditions, $a_1 \sim 0.2$ and $\alpha \sim 0.1$. Since this may not be true for other conditions, we discuss $\cap$ in detail below. However, here we propose that this sort of proportionality extends to shear; that mechanical contributions to a shear inclusive turbulent scaling velocity, w_{*s}, may be written in terms of the boundary layer integrated mechanical generation rate of TKE. I.e.,

$$w_{*s} \simeq \left[\frac{\int_{z0}^{H} \beta\overline{w'\theta'}_z \, \partial z + \int_{z0}^{H} -\overline{u_i'w'}u_{i,z} \partial z}{\cap} \right]^{1/3} \quad . \tag{5}$$

fig.1 Schematic profiles of a) buoyancy, b) stress, c) U velocity, and d) TKE shear generation rate for barotropic case



We also propose that the CBL shear generation rate can be represented by discrete barotropic surface layer (SL), entrainment zone (EZ), and baroclinic (BC) components.

$$\int_{z0}^{H} \overline{-u_i'w'}u_{i,z}\partial z = \overset{SL}{\int_{z0}^{-L} \overline{-u_i'w'}u_{i,z}\partial z} + \overset{EZ}{\int_{zi}^{H} \overline{-u_i'w'}u_{i,z}\partial z} + \overset{BC}{\int_{0}^{H} \overline{-u_i'w'}u_{ib,z}\partial z} \quad . \tag{6}$$

Here $u_{ib,z}$ is just the baroclinic portion of the vertical shear.

BAROTROPIC SURFACE LAYER SHEAR

We first consider the barotropic SL and assume that most of the shear occurs below the height, -L. From SL similarity we have

$$u_{,z} \simeq (u_*/kz)\phi_m \quad , \quad \text{with} \quad \phi_m \simeq (1 - 15z/L)^{-1/3} \quad , \tag{7a,b}$$

(Carl et al. 1973). Some workers prefer ϕ_m formulas involving the -1/4 power. However, the choice is among empirical expressions and integrals of -1/4 power expressions result in awkward complex numbers. The accepted value in the free convection limit is -1/3, so we choose this for theoretical appeal as well as mathematical convenience.

Since ϕ_m is only valid for $z >> z_0$, we set the upper integration limit at -L, the lower limit at a somewhat arbitrary 10 z_0, and treat the layer below 10 z_0 separately. Above 10 z_0, from eqns. (2b) and (7a), we have

$$\int_{10z0}^{-L} \overline{-u_i'w'}u_{i,z}\partial z \simeq \frac{u_*^3}{k}\int_{10z0}^{-L} \frac{\phi_m\partial z}{z} + \frac{u_*(u_*^2 - \Delta u_iWe)}{kz_i}\int_{10z0}^{-L} \phi_m\partial z \quad ,$$

$$\simeq \frac{u_*^3}{k}\int_{10z0}^{-L} \frac{\phi_m\partial z}{z} - (1-a_2)(L/z_{i2})w_*^3 \int_{10z0}^{-L} \phi_m\partial z \quad . \tag{8}$$

From the L definition, $u_*^3/k = (-L/z_i)w_*^3$. With the axes aligned with the mean wind, we have $-u'_iw'u_{,z} \simeq -u'w'u_{,z}$, since $(v_{,z}, w_{,z}) << u_{,z}$ in the mixed layer. If shear supplies sufficient TKE, 10 z_0 <<< -L. So in eqn. (8) term (2) $\simeq 0.54\ (1 - a_2)\ (L/z_i)^2w_*^3$. Letting $p = -15z/L$, the first integral reduces to

$$\simeq \sqrt{3}\ \mathrm{ATAN}\left[\frac{\sqrt{3}(2(p+1)^{1/3}+1)}{3}\right] - \frac{\mathrm{Ln}[(p+1)^{2/3}+(p+1)^{1/3}+1]}{2} +$$

$$\mathrm{Ln}[(p+1)^{1/3}-1]\Big|_{10z0}^{-L} .$$

The upper limit evaluates to $\simeq 1.51$. The first two terms in the lower limit $\simeq 1.26$, if $L >> 150z_0$. So in eqn. (8) the first integral $\simeq 0.25 - \mathrm{Ln}\ R$, where $R \equiv (1 - 150z_0/L)^{1/3} - 1 << 1$. A 5cm z_0 and 150m L would give $150\ z_0 = -0.05L$. Above 10 z_0, this leads to $\simeq 4.36u_*^3/k$ from term (1) in eqn. (8). Oddly, R varies inversely with roughness length. But from $u_* = ku/(\ln(z/z_0 - \varphi_m)$ and the definition of L, we see that

$$nz_0/L = (ng\overline{w'\theta'}/k^2u^3\theta)z_0(\ln(z/z_0) - \varphi_m)^3 \quad . \qquad (9)$$

So for given levels of u, $\overline{w'\theta'}$, θ, and z, the ratio nz_0/L and thus term (1) may vary much less than z_0 itself.

Below 10 z_0 we neglect non-neutral contributions to u_* and assume a constant value, $-u_i'w'u_{,z} \simeq u_*^3/10kz_0$. Then

$$\int_0^{10z0} -\overline{u_i'w'}u_{,z}\partial z \approx u_*^3/k \quad . \qquad (10)$$

Together, eqns. (8) and (10) give an estimate of the vertically integrated SL shear production rate,

$$\int_0^{-L} -\overline{u_i'w'}u_{,z}\partial z \simeq (1.25 - \mathrm{Ln}\ R)u_*^3/k \quad ,$$

$$\simeq (-w_*^3L/z_i)(1.25 - \mathrm{Ln}\ R) \quad . \qquad (11a,b)$$

Note that we have dropped term (2) from eqn. (8), since 1) the coefficient is an order of magnitude smaller than for term (1), 2) term (2) is second order in z_i, 3) usually, $-L < 0.4\ z_i$, and 4) a_2 is of order 10^{-1} - 1; so some further cancellation occurs.

From eqns. (2) and (3), if $\cap \simeq 0.4$, we have the following ratio of enhanced to unenhanced TKE contributions,

$$\left[\frac{\cap - (1.25 - \mathrm{Ln}\ R)L/z_i}{\cap}\right]^{1/3} w_* \simeq (1 + (3.1 - 2.5\mathrm{Ln}\ R)(-L/z_i))^{1/3} w_* \ . \tag{12}$$

For our previous example, w_{*s} exceeds w_* by a factor of $\simeq (1 + 33u_*^3/w_*^3)^{1/3}$. In this case, since $u_*/w_* \simeq 0.54$, SL shear generation alone may enhance w_{*s} by more than 80%.

To be useful, w_{*s} should maintain the same proportion to the mean turbulent velocity, $\sigma_{ui}/3$, in both the convective and neutral limits. We test this by noting for the barotropic case that the cube root of the vertically integrated production rate is usually $\simeq 2.4\ u_*$ for SL shear. The EZ and BC terms are not relevant for the pure neutral case. From Grant (1986) and Mason and Thompson (1987), we find that $\sigma_{ui}/3 \simeq 1.3\ u_*$ in the layer averaged neutral boundary layer. This implies that $\sigma_{ui}/3 \simeq 0.53\ w_{*s}$. In the pure convective limit, the CBL averaged value for $\sigma_{ui}/3$ is $\tilde{}\ 0.55\ w_*$ (Caughey and Palmer, 1979 and Deardorff et al. 1980). So, the ratio of w_{*s} to the layer averaged mean turbulent velocity is the same in the convective and neutral limits, as required.

ENTRAINMENT ZONE SHEAR

The EZ also generates mechanical turbulence. To quantify it we assume, moreover, that shear is constant across Δh. Thus,

$$\int_{zi}^{H} -\overline{u_i'w'}u_{i,z}\partial z \simeq We\Delta u_i^2/2 \quad . \tag{13}$$

Equation (13) suggests the need for a simple shear inclusive model to estimate the entrainment rate, We. For this purpose we parameterize the TKE equation,

$$\underbrace{e_{,t}}_{\text{storage}} = \underbrace{\beta\ \overline{w'\theta'}}_{\text{buoyancy}} + \underbrace{(\overline{u_i'e} + \overline{p'e}/\rho)_{,z}}_{\text{transport}} - \underbrace{\overline{u_i'u_j'}u_{i,j}}_{\text{shear}} \underbrace{-\nu\overline{u_{i,j}u_{i,j}}}_{\text{dissipation}} \quad , \tag{14}$$

at the inversion base as,

$$\underbrace{C_{st}w_*^2We/\Delta h}_{\text{storage}} \simeq \underbrace{-\beta\Delta\theta We}_{\text{buoyancy}} + \underbrace{C_c w_*^3/z_i}_{\text{transport}} + \underbrace{C_m\Delta u_i^2 We/\Delta h}_{\text{shear}} - \underbrace{C_d w_*^3/z_i}_{\text{dissipation}} \quad . \tag{15}$$

The scaling arguments are that 1) again $\overline{w'\theta'}_{zi} \simeq -We\Delta\Theta$ and 2) large mixed layer eddies drive the TKE transport rate into the EZ. Thus TKE transport should vary with the TKE ($\propto w_*^2$) times the large eddy turnover time scale, z_i/w_*. 3) The shear production rate is modeled as the momentum flux, $\Delta u_i We$, times the local EZ shear, $\Delta u_i/\Delta h$. 4) To maintain a steady state inertial subrange, the dissipation plus buoyancy destruction rates must balance the large eddy TKE injection rate due to transport and shear production. So in the neutral limit, dissipation should scale like transport with the change rate of large eddy TKE, w_*^3/z_i. Since dissipation occurs at small scales, unaffected by buoyancy anisotropy, we retain the neutral form. For simplicity we neglect the small TKE storage rate term. C_m estimates vary somewhat, (Pollard et al. 1981; Kato and Phillips, 1967, Tennekes and Driedonks, 1981), but we see little reason for departures far from unity, so we assume $C_m \sim 1.0$. We discuss C_c and C_d below.

Tennekes and Driedonks (1981) presented a similar model, except that here we scale the shear production locally with Δh instead of z_i. Solving for the entrainment rate gives,

$$We = \frac{(C_c - C_d)}{\beta\Delta\Theta - \Delta u_i^2/\Delta h} w_*^3/z_i \quad . \tag{16}$$

Therefore, from eqn. (13) we have

$$\int_{zi}^{H} -\overline{u_i'w'}u_{i,z}\partial z \simeq \frac{(C_c - C_d)}{2(R_b - 1)} \frac{\Delta h}{z_i} w_*^3 \quad , \tag{17}$$

where the bulk Richardson number,

$$R_b \equiv \frac{\beta\Delta\Theta\Delta h}{\Delta u_i^2} \quad . \tag{18}$$

The EZ is transitionally turbulent by nature. Thus, we expect R_b to fluctuate about some critical threshold turbulence value, R_{cb}. R_{cb} is much larger than the 1/4 value accepted for the stable SL because EZ TKE is mainly maintained by large eddy transport from below. So shear generation is only part of the TKE injection rate, and the dominant eddies are also much larger than in the transitionally turbulent SL. Thus, the EZ's large negative buoyancy will destroy much of the TKE during the eddy cascade before it can be dissipated at small scales. For these reasons

R_{cb} may exceed unity in the EZ.

Indeed, our simulations to steady state using such an entrainment model suggest a typical value of $R_{cb} \simeq 1.3$. From the results of Turner (1968), Deardorff et al. (1980), and many others, we suggest that $C_c - C_d \simeq 0.25$ in the convective limit. So in eqn. (17) the EZ shear contribution $\simeq 0.5\ \alpha w_*^3$, where $\alpha \equiv \Delta h / z_i$.

To assume that $\alpha \simeq 0.1 - 0.2$ may often suffice. Then, $a_l \simeq 0.2$, $\cap \simeq 0.4$, and the barotropic EZ contribution to w_{*s} is only a few percent. But α can exceed unity during rapid entrainment into a near-neutral layer remaining from the previous day's CBL (Nelson et al. 1989). During such times, entrainment models suggest that a_l may exceed 40% (Kamada, 1988a). Then $\cap \simeq 0.1$, and EZ enhancement of w_{*s} may exceed 0.4. With SL shear this may cause w_{*s} to double. In this case a better diagnostic or prognostic estimate for α is needed. For a diagnostic estimate, we begin by assuming $R_b = R_{cb}$ and solve eqn. (17) for Δh. This leaves unknown Δu_i and $\Delta\theta$, the inversion temperature jump. $\Delta\theta$ is hard to specify more precisely than ~ +/- 30% because z_i and H are not that highly resolved. Yet, we know that $\Delta\theta$ grows with the boundary layer growth rate, $H_{,t}$, into the layer aloft (with lapse rate Γ_u), and shrinks with the mixed layer warming rate, $\Theta_{m,t}$. So, using the simplest steady state assumption,

$$\Delta\theta_{,t} = \Gamma_u H_{,t} - \theta_{m,t} = 0 \quad . \qquad (19)$$

The mixed layer warming rate is given by the surface heating rate, plus the entrainment rate of warm air from aloft. i.e.,

$$\Theta_{m,t} = (\overline{w'\theta'}_0 - \overline{w'\theta'}_{zi})/z_i \quad . \qquad (20)$$

So, if we approximate $H_{,t}$ as $We(1 + \alpha)$, we have

$$\Delta\theta = \overline{w'\theta'}_0/We - \Gamma_u z_i(1 + \alpha) \quad . \qquad (21)$$

Using eqns. (16) and (18) and combining constants, this results in two solutions for α. We show only the physical root,

$$\alpha = \left[\frac{2.3\Delta u_i^2}{\Gamma_u\ \beta\ z_i^2} + \frac{1}{4}\right]^{1/2} - \frac{1}{2} \quad . \qquad (22)$$

Though not explicit, eqn. (22) implicitly involves $\Delta\Theta$ because it presumes through eqns. (18 - 22) that $\Delta\Theta$ depends on Γ_u, We, $\overline{w'\theta'}_0$, and z_i. This assumes that the capping inversion requires finite potential temperature and windspeed jumps, whenever there is horizontal wind.

In eqn. (22) we must still estimate the windspeed jumps across the EZ. Wyngaard (1988) proposed the following first order estimate. For steady flow over a horizontally homogeneous surface, the stress gradient is given by

$$-\overline{u'w'}_{,z} = f(v - v_g) \quad , \quad \text{and } -\overline{v'w'}_{,z} = f(u_g - u) \quad , \qquad (23a,b)$$

where f refers to Coriolis forcing, and the subscript, g, refers to geostropic winds (Panofsky and Dutton, 1984). For barotropic flow with the axes aligned with the mean mixed-layer wind,

$$-\overline{u'w'}_{,z} = fv_g \simeq (We(u_g - u) + u_*^2)/z_i \quad , \quad \text{and}$$

$$-\overline{v'w'}_{,z} = f(u_g - u) \simeq We(v_g - v)/z_i \quad . \qquad (24a,b)$$

The momentum balance becomes

$$We(u_g - u) + u_*^2 = -fz_i v_g \quad , \quad \text{and}$$

$$-We\ v_g = fz_i(u_g - u) \quad . \qquad (25a,b)$$

For barotropic cases, if $\Delta u \simeq (u_g - u)$ and $\Delta v \simeq (v_g - v)$ across the EZ, then the solutions to eqns. (25a,b) are to first order,

$$\Delta u = We(u_*/fz_i)^2 \quad , \quad \text{and}$$

$$\Delta v = u_*^2/fz_i \quad . \qquad (26a,b)$$

Here, almost all of the EZ shear results from turning the wind into the v direction. So for a mature mid-latitude BL with $u_* = 0.4 ms^{-1}$, $f = 10^{-4}s^{-1}$, $z_i = 10^3 m$, $\Theta = 300$ °C, and $\Gamma_u = 3 \times 10^{-3}$ °C m^{-1}, we have $\alpha \simeq 0.057$. However, during growth into the near neutral remnant mentioned earlier, conditions may be more like $\Gamma_u = 1 \times 10^{-3}$ °C m^{-1}, and $z_i = 500m$. This would result in $\alpha \simeq 1.3$. Values spanning such a range were reported in Nelson et al. (1989). They also observed and prognostically modeled the typical early morning to late afternoon hysteresis found in plots of α versus We/w_*. They noted that such hysteresis is absent from previous diagnostic expressions for the EZ depth which have the form $\alpha \propto Ri_*^{-n}$, where $Ri_* \equiv \beta z_i \Delta\theta / w_*^2$ and $1/4 \le n \le 1$. No hysteresis occurs in such expressions, if we assume (see eqn. 2a) that a_l, the inversion to surface heat flux ratio, is constant because then $We/w_* \propto 1/Ri_*$. Unlike such expressions, eqn. (22) shows qualitatively appropriate hysteresis. In fact, by combining eqns. (16), (21), (22), and (26), we see that

$$\frac{\alpha}{We/w_*} \simeq \frac{u_*^4 w_*}{w'\theta'_0 \beta f^2 z_i^3} \quad . \tag{27}$$

Far from being constant, $\alpha w_*/We$ will typically increase initially as surface heating and u_* grow in the young day-time boundary layer, but will diminish later as z_i becomes large.

If we require more accuracy, we can couple eqns. (16, 18, 19, 20, and 26) with

$$H_{,t} \simeq We + (R_{cb}/\beta)\left[\frac{2u_*^2 \Delta v\ We}{f z_i^2 \Delta\theta} - \frac{\Delta v^2 \Delta\theta_{,t}}{\Delta\theta^2}\right] \tag{28}$$

to provide a complete entrainment model. Here the second term in eqn. (28) gives the growth rate of Δh and thus a prognostic estimate for α.

More physics may be added by revising the We expression to include the small TKE storage rate term, $C_{st} w_*^2/\Delta h$, from eqn. (16). However, our present focus is amending w_*, rather than refining the particulars of bulk entrainment models; so we choose to rest with the above diagnostic and prognostic formulations for α.

In either case, hysteresis in α versus We/w_* is evident, and like Kamada (1988b, figs. 1 and 2), depends on changes in $\Delta\theta$ and z_i. Both models include shear as well as heating due to entrainment, and avoids ad hoc surface temperature distributions, or fixed values of a_l. Also note that eqns. (16) and (21) lets us specify a_l which, together with eqn. (22), determines $\cap$ from eqn. (4).

BAROCLINIC SHEAR

If the geostrophic wind varies linearly with height according to the thermal wind relations, $u_{g,z} = -(g/fT_0)T_{,y}$, and $v_{g,z} = (g/fT_0)T_{,x}$, then from eqn. (23) the baroclinic stress profile must be parabolic, rather than linear as in the barotropic case. Wyngaard (1985) gave the following simple estimate for baroclinic shear based on the thermal wind,

$$u_{b,z} = b_1 u_{g,z} + b_2 v_{g,z} \quad , \quad v_{b,z} = b_2 u_{g,z} + b_1 v_{g,z} \quad , \tag{29a,b}$$

where $b_1 = 1/(m^2 + 1)$, $b_2 = m/(m^2 + 1)$, $m = fz_i/w_*$, and f is the Coriolis forcing. If so, the stress can be written as

$$-\overline{u_i'w'} = u_*^2 + \frac{b_2 z\ (gz\ \rho f T_{,y} + 2\ fT\ p_{,y})}{2\ \rho f T} \quad . \tag{30}$$

So the baroclinic shear contribution becomes

$$\int_0^H -\overline{u_i'w'}u_{i,z}\partial z \simeq \frac{gz_i(b_2 T_{,x} - b_1 T_{,y})(b_2 g\rho z_i^2 T_{,y} + 6\rho T u_*^2)}{6\ \rho f T^2} \quad , \tag{31}$$

where terms in $b_1 - 1 = -m^2/(m^2 + 1)$ were considered neglectably small. The baroclinic integral is taken over the whole vertical range 0 to z_i rather than -L to z_i because the SL shear analysis only accounts for barotropic effects based on SL similarity. Scaling analysis of eqn. (31) shows that the squared and cubic terms in z_i are small. If so, to first order, the baroclinic shear contribution from eqn. (31) reduces to,

$$\int_0^H -\overline{u_i'w'}u_{i,z}\partial z \simeq (z_i/z_b)w_*^3 \quad , \tag{32}$$

where we can define a baroclinic length scale as,

$$z_b \equiv \frac{fT\ w_*^3}{u_*^2 g(b_2 T_{,x} - b_1 T_{,y})} \simeq \frac{w_*^3}{u_*^2\ u_{b,z}} \quad . \tag{33}$$

Usually, $z_b >> z_i$. But in coastal areas mesoscale temperature and pressure gradients are often large. For a coastal z_i of 300 m, a 3 °C/100 km horizontal cross-wind temperature gradient, We near zero, $f = 10^{-4}$ sec^{-1}, $w_* \simeq 0.8$ m s^{-1}, and $u_* \simeq 0.4$ m s^{-1}, then $z_b \simeq$ 330 m, and w_{*s} may be enhanced baroclinically by ~ 50%.

SUMMARY

In summary, we account for surface layer (SL), entrainment zone (EZ), and baroclinic (BC) shear. Each source is given in terms of a dimensionless ratio of the relevant length scales z_0, L, Δh, z_i, and z_b. When summed and multiplied by w_*, this results in a turbulent velocity scale which includes both buoyancy and shear generation of TKE, namely,

$$w_{*_s} \equiv \left[\frac{\cap - (1.25 - \mathrm{Ln}\, R)L/z_i + 0.5\Delta h/z_i + z_i/z_b}{\cap} \right]^{1/3} w_* \quad . \tag{34}$$

Here $\cap \equiv \dfrac{1 - a_1(1 + \alpha)}{2}$,

$a_1 \simeq We\Delta\theta/\overline{w'\theta'}_0$, the z_i/surface heat flux ratio,

$We \simeq \dfrac{(C_c - C_d)}{\beta\Delta\theta - \Delta u_i^2/\Delta h} w_*^3/z_i$, the ML entrainment rate,

$\Delta\theta \simeq \overline{w'\theta'}_0/We - \Gamma_u z_i(1 + \alpha)$, the EZ θ jump,

$\alpha \equiv \Delta h/z_i \simeq \left[\dfrac{2.3(\Delta u^2 + \Delta v^2)}{\Gamma_u \ \beta \ z_i^2} + \dfrac{1}{4} \right]^{1/2} - \dfrac{1}{2}$, the EZ/BL depth ratio,

$\Delta u \simeq We(u_*/fz_i)^2$, $\Delta v \simeq u_*^2/fz_i$, the EZ velocity jump,

$R \equiv (1 - 150z_0/L)^{1/3} - 1$, an SL shear generation term,

$z_b \equiv \dfrac{w_*^3}{u_*^2 \ u_{b,z}}$, the baroclinic length scale.

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